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INITIAL STATE SPACE APPROACH TO ANALYSIS IN
STUDIES USING AMSWAG

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20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*
This study presents the development and use of a surrogate model for estimating wargame outcomes from only the specification of the conditions, or initial "state," which exists at the start of the engagement.

The methodology has been found useful in real world problems in the fields of medicine, chemistry, etc. It is based upon a multivariate regression or response surface approach although some additional refining techniques such as cluster analysis are also used. The utility of the current results and areas

20. for future work are discussed.

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(5) to be used as a surrogate for the original model in a larger scale model or simulation.

The general methodology may be termed the "state space" approach to modeling, where the term state refers to the values of the input variables. In the next two sections of this report, recent applications of this approach to other problems are described. The discussion then turns to the application of the methodology to the evaluation of the results obtained from the AMSWAG[1,2], a combat simulation.

1.2 Previous Applications of State Space Approach to Modeling Simulations.

In this section a brief review of the application of state space methodology (SSM) and pattern recognition (PR) to several DoD problems is presented. In the first application we describe the use of SSM and PR in obtaining trauma indices based on physiological, biochemical, and anatomical measurements taken from trauma patients at several medical centers.

The second application consists of rationales and computer techniques for characterizing, clustering, and screening chemical compounds for potential biological activity and for relating structural properties of compounds to pharmacological activities. Finally, the application of the approach to estimating the output of a weapon system performance model is summarized.

1.2.1 Trauma Indices. Considerable effort has been devoted to the development of a broad class of trauma indices covering a range of patient conditions. The original work was begun in 1973, a joint venture among the Biophysics Branch of the Chemical Systems Laboratory, AMSAA, and the Maryland Institute for Emergency Medical Services (MIEMS). Efforts have continued in MIEMS and also with surgeons at Washington Hospital Center and Monmouth Medical Center.

The indices evolved from pattern recognition analyses of over 60 physiological and biochemical variables. Each of the individual variables, and many combinations of variables, were evaluated using methods which assess their capability to independently and correctly predict patient outcome, i.e., survival or death.

On the basis of these computations, advice from clinicians, and practicality, the indices were derived. In each case the index -- which is a function of one or more physiological and biochemical variables -- was used to characterize the "state" of the patient. For each of the indices, a probability of mortality curve was obtained by fitting the data to a logistic model.[3] Approximate 95 percent confidence bounds on the curve were computed by the method of Kendall and Stuart.[4]

For example, a Respiratory Index (RI) was developed as an indicator of a trauma patient's respiratory state[4]. A Renal Index (REI)

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was developed as an adjunct method for evaluation of renal (kidney) failure and to give indications of the need for early hemodialysis in trauma patients.[5] An Acute Trauma Index (ATI) and a Blunt Anatomical Index (BAI) were developed to characterize patient status at the time of admission to a hospital.[6, 7] A Triage Index has also been developed.[8] It is a validated technique for an early, rapid, noninvasive, accurate method for estimating injury severity, permitting appropriate matching of trauma victims with available therapeutic resources as a means of reducing mortality and morbidity.

Mortality correlations are available for all of the indices individually and for certain combinations of indices in order to accurately characterize the illness or injury to multiple body subsystems or to characterize the physical and biochemical states of the patient.

All of the indices are currently being used in various applications by researchers and clinicians at the MIEMS, Washington Hospital Center, Monmouth Medical Center and hospitals throughout the United States.

The applications of these indices include patient triage (which could be most useful in military combat situations), prognosis (at the time of hospital admission and throughout the patient stay) and tracking of patient condition; initiation, assessment and communication of therapies; and general evaluation of care.[5, 9-17]

1.2.2 Screening and Structure-Activity Studies of Chemical Compounds. During the past several years, a multidisciplinary group including biochemists, mathematicians, statisticians, and computer scientists from several elements of the Chemical Systems Laboratory, ARRADCOM, has been applying pattern recognition techniques to the screening of chemical compounds and to modeling the structure-pharmocological relationships of several classes of compounds.

The methodological developments include rationales and computer techniques for characterizing, clustering, and for screening chemical compounds for potential biological activity, and for relating structural properties of compounds to pharmacological activities.

In all of these applications a compound is characterized by a property vector $X = (x_1,..., x_n)$ where x_i is a number which corresponds to the value of the i-th property. A property may be any characteristic which is believed to have some relationship to a pharmacological activity of interest; it may be a physiochemical property such as molecular weight or partition coefficient; a steric property such as bond radius or an interatomic distance; or an arbitrarily chosen structural property such as the number of oxygen atoms or number of occurrences of a given substructure.

In a number of applications clustering has been used as a basis for screening a set of candidate drugs for testing. In these applications,

the n dimensional property vectors of the compounds are clustered and several drugs are selected from each cluster for testing. The objective is to select a small set of drugs which are "representative" of all candidates. The methodology is based on the premise that drugs which are "close" to one another in property vector space will have similar activities. As activity data become available, the measure of "distance between drugs" can be altered to reflect the relative "prediction of activity" capabilities of individual properties of the property vector and an index to predict activity can be constructed.

1.2.3 Response Surface Methodology Application to the Laser Designator Weapon System Simulation (LDWSS). LDWSS is a detailed stochastic simulation model which was developed at MICOM in order to guide the development and aid in the evaluation of laser guided weapon systems such as COPPERHEAD and HELLFIRE. Because of the relatively high cost of exercising the LDWSS, AMSAA is conducting an investigation to generate, through response surface or regression techniques, a mathematical model for estimating the results of the LDWSS. The effort has produced a low-cost, quick-response method for estimating LDWSS results which could be included in combat simulations which consider semi-active, laser-guided weapons such as COPPERHEAD or HELLFIRE and yet cannot, due to time constraints, include LDWSS as a subroutine for making effectiveness computations.

Data that were extracted from 336 LDWSS runs simulating COPPERHEAD firings have been subjected to a cluster analysis. This process aids in defining void regions in the input data space and magnifies the benefits obtained from the regression analysis. After using an auxiliary program to sort the cases according to cluster membership and to calculate within-cluster statistics, regression models were developed for each of the clusters as well as for the complete set of 336 cases.

Some recently generated LDWSS COPPERHEAD cases were obtained to validate the regression models developed. The estimates of probability of hit obtained from those models agreed rather well with those generated by the LDWSS. Current efforts are focused on analyzing several hundred recently generated LDWSS COPPERHEAD cases to be added to the somewhat small data base of 336 cases.

2. DISCUSSION

The previous section briefly described a few applications of the state space approach to the modeling of "systems"; human, chemical and one simulation. The objective of this report however, is to describe the application of this methodology to a combat simulation frequently used by AMSAA to evaluate tactics and materiel. AMSWAG is briefly described in the following section.

2.1 Description of the AMSWAG Model.

AMSWAG is a computerized combat simulation which describes a typical battalion level attack/defense situation. The model is deterministic, time-sequenced and is based on second order differential equations.

The defending force, normally a reinforced company, is deployed in a fixed position. The attacking force, normally a battalion, moves along predefined paths of advance. Each defender and attacker unit consists of a homogeneous group of weapons, such as M60A3 tanks.

AMSWAG "conducts" the battle in uniform time steps of ten seconds each. The primary processes considered during each interval are target acquisition, target prioritization, target allocation, fire suppression, attacker dismount, and target attrition. Attrition is due to direct fire, indirect fire (artillery and mortar), and minefields, although it must be admitted that these threats are played less than perfectly. At the end of each time step, the number of survivors in each unit is determined by subtracting the attrition to the unit; the ammunition levels are also depleted appropriately. AMSWAG also provides a description of total vehicle and personnel losses on each side, vehicle exchange and force ratios, status of surviving units, and killer-victim scoreboards (number of kills as a function of weapon type versus weapon type). The normal stopping rule for an AMSWAG battle is a specified level of losses, usually 60 percent for either the attacker or defender.

2.2 Application of State-Space Approach to Modeling AMSWAG.

We begin the application by choosing the variables which will be used to characterize the initial "state" of the AMSWAG engagement. This was achieved subjectively based upon experience with the model. The data which formed the basis for analysis thus consisted of one "state" vector per game and the associated values of one or more output measures of interest. The next step was to determine the relationship between the state variables and the output variables.

Two analyses of AMSWAG results were performed. The first used the data from 35 AMSWAG runs performed to support the Engineer Study Phase I. The second analysis utilized 155 AMSWAG runs, including 120 new cases from the Phase II Engineer Study.

The following section describes the input variables used in the analysis.

2.2.1 Input/Output Variables of Interest. State variables
used in the analyses of the 35 runs were, defender's exposure (E), minefield density (M), time-frame (T), preparation time (P), and the attacking forces countermeasure (C). Each of these state variables is described in the following paragraphs.

The exposure of a defender target (E) is represented by a coded variable whose values range from 1 to 4. An index value of 1 means that the defender is fully exposed (FE). Index values of 2, 3, or 4 mean the defender is in 1/3, 2/3, or full hull (HD) defilade respectively. Exposure obviously affects the probability that a defender will be hit by enemy fire and hence his rate of attrition.

Minefield density (M) is a coded variable which describes a combined density of both remote emplaced (RAAMS) and ground emplaced (GEMSS) systems. The density was a measure of mines per square meter.

Time-frame (T) is also a coded variable which takes on one of the values 1, 2, or 3, which are descriptive of the current (1978), future time-frame (1982), or future time-frame with XM1 respectively. This variable is important as much of the detailed weapon performance, vulnerability, etc. depend on time-frame. In a sense then, time-frame is a surrogate for those detailed data.

Preparation time (P) reflects the amount of warning time available to the defender force of an impending attack. During this period the defender allocates his resources to either the preparation of weapon positions or the implacement of barriers. The preparation of weapon positions is equivalent to gaining decreased exposure. The emplacement of barriers includes minefields and nondestructive barriers such as an abatisse or tank ditches. Preparation time takes on the values 1, 2, 3 and 4 for 0 hours, 6 hours, 24 hours, and greater than 24 hours warning times respectively.

Attacking force countermeasure (C) is a coded variable which describes the tactics used by the attacker upon encountering a minefield. The values of C range from 1 to 4, indicating a bull-through, line-breach, column-breach, or delay tactic respectively. During a bull-through, the attacker proceeds through the minefield as if there were no minefield. In a line-breach, individual columns follow a plow vehicle through the minefield. When employing a column-breach, the individual attacker units on different routes through the minefield, converge into one column behind a plow vehicle. After exiting the minefield units again return to their original routes. During a delay tactic, the attacker either delays his advance before or from within a minefield.

Output variables of interest included: attacker win or loss, time till the end of the game; and final force ratios (attacker/defender) for both vehicles and personnel.

Attacker win or loss is a coded output variable determined from the AMSWAG game results. An attacker win is attained when the defending force has been attrited to 60 percent of its original strength. Similarly, an attacker loss is defined to occur when the attacking force has been attrited to a similar degree. The variable is coded 1 or 0 for an attacker win or loss respectively.

2.2.2 Application of Approach to 35 AMSWAG Games. The 35 simulation runs of the Engineering Study Phase I provided the basis for the analysis described here. The following 5 variables were used to describe the state for each of these 35 games:

(1) minefield density (M),

(2) time-frame (T),

(3) preparation time (P),

(4) attacking force countermeasure (C), and

(5) exposure (E).

2.2.2.1 PER Methodology (Phase 1 Study). The first output variable analyzed was attacker win or loss, which was coded 1 or 0 respectively. The value of each of the state variables for predicting outcome was determined using the PER methodology. When both the dependent and independent variables are continuous, correlation is often used to make such assessments. However, in this case the dependent variable, outcome, is two-valued, hence the use of correlation is inappropriate. Instead other measures based upon information theoretic concepts were used. These measures may be symbolized by the acronym PER.

In this application P stands for the a priori probability of attacker win, i.e., the percent of the 35 cases which were "won" by the attacker. Then for any state variable x we can compute quantities E_x and R_x which are respectively called the information gain and relative information gain provided by x. Appendix A gives a thorough discussion of PER. It suffices to say here that the value of E_x depends upon P (namely $0 \leq E_x \leq 2P(1-P)$) and that to remove this dependence R_x is defined as:

$$R_x = \frac{E_x}{2P(1-P)} \text{, hence } 0 \leq R_x \leq 1.$$

The larger the value of R_x, the better the variable x is at predicting outcome.

For the 35 AMSWAG cases considered:

$$P = 25/35 = .71 = \frac{\text{Number of games won by attacker}}{\text{Total number of games}}$$

Table 1 lists the input variables in relation to P, E_x, R_x.

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TABLE 1 RESULTS OF PER METHODOLOGY ON 35 AMSWAG CASES

	P	E_x	R_x
EXPOSURE (E)	.71	.28	.68
MINEFIELD DENSITY (M)	.71	.25	.61
TIME-FRAME (T)	.71	.26	.63
PREPARATION (P)	.71	.20	.49
COUNTERMEASURE (C)	.71	.16	.34

Results indicated that for the 25 engagements out of 35 where the defender's exposure (E) was in any state but full hull defilade and the time-frame (T) was current, the attacker always won. The attacking force lost only when the defender was in a state of full hull defilade in the future time-frame.

Referring to Table 1, it should be noted that the variables exposure (E) and time-frame (T) provide the highest values of E_x and R_x. Listed below are the results of a PER assessment made of the variables exposure (E) and time-frame (T), when considered jointly:

	P	E_x	R_x
(Exposure, time-frame)	.71	.39	.95

For the 35 case study it was found that the 5 initial input variables could essentially be reduced to 2, i.e., exposure and time-frame, in order to predict win or loss with a high degree of certainty. This is only true for this limited data base and should not be construed as a general result.

In the following section a more general approach to predicting outcome is explained and applied to our 35 game data base.

2.2.2.2 Regression on Win or Loss (Phase 1 Study). The logistic function, which is a nonlinear functional form, is often used to obtain estimates of a probability from multiple inputs when outcomes are two-valued such as win or loss.

The goal of the logistic function is to estimate the coefficients of a polynomial $A(x)$ (where $A(x) = A_0 + A_1 X_1 + A_2 X_1, + \ldots A_m X_m$) which can be used to predict a probability, in this case of "attacker win," $P(X)$. A_1, A_2, ...,A_m are the coefficients associated with the independent variables X_1, X_2, ..., X_m.

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In general, vectors comprised of the initial state variables and their respective game outcomes are entered into an iterative least-squares solution.[14] This process continues recursively until coefficients are obtained for the input variables to provide estimates of the probability of the attacker win.

With the estimated coefficients, P(X) is determined as:

$$P(X) = \frac{1}{1 + e^{-A(x)}}.$$

See Figure 1 for a graphical representation of this function.

In order to demonstrate the use of the logistic function for the 35 game study, a simple linear model was chosen for A(x), in which each of the five variables entered linearly. The following polynomial resulted for the 35 AMSWAG cases:

A(x) = 711 - 94 (exposure) - .0039 (minefield density) - 263 (time-

frame) - 10.4 (preparation time) + 593 (countermeasure)

Table 2 lists the relationships of the independent variables, A(x), and the response P(x), i.e., probability of an attacker win, for the 35 AMSWAG games on which the analyses were based. The coefficients listed above represent the importance of each variable to outcome. It can be seen that the attacker's countermeasure (C) is most important to his winning. Exposure (E) and time-frame (T) are most beneficial to the defender.

It may be noted that while C has the largest coefficient, it has the smallest relative information gain (Table 1) of any variable. This anomaly could be caused by several factors. First, the PER approach considers each variable in isolation, while the regression considers all variables simultaneously, adjusting coefficients so as to achieve the best fit. Secondly, as noted earlier in this section, quite good prediction could have been achieved using only the variables Exposure and Time Frame. While all five variables were used in the model for demonstration purposes, this does permit the adjustment of coefficients on all variables to achieve the best possible fit, in this case causing some unexpected results.

From Table 2 it should be clear that for the 35 AMSWAG cases, the logistic function provided a perfect prediction of final outcome. The probability of the attacker winning is a clear win or loss, indexed 1 or 0 respectively. Figure 2 is a graphical representation of how the logistic function appears for the 35 case study.

The logistic function was also used to predict the outcome for games not yet played, but which may be described by allowing the input variables to vary within their respective ranges. The logistic function

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FIGURE 1   GRAPHICAL REPRESENTATION OF THE LOGISTIC FUNCTION

$$P(x) = \frac{1}{1 + e^{-A(x)}}$$

Where $A(x)$ is a polynomial.

(1) In its simplest form with $A(x) = x$, we have
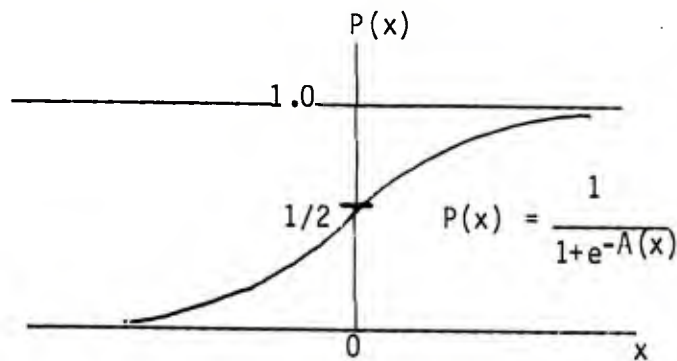
$$P(\dot{x}) = \frac{1}{1 + e^{-x}} \quad .$$

(2) $A(x)$ could be a linear combination of several variables –

$A(x) = A_0 + A_1 X_1 + A_2 X_2 + \ldots + A_n X_n.$

(3) $A(x)$ could be a polynomial of one variable –
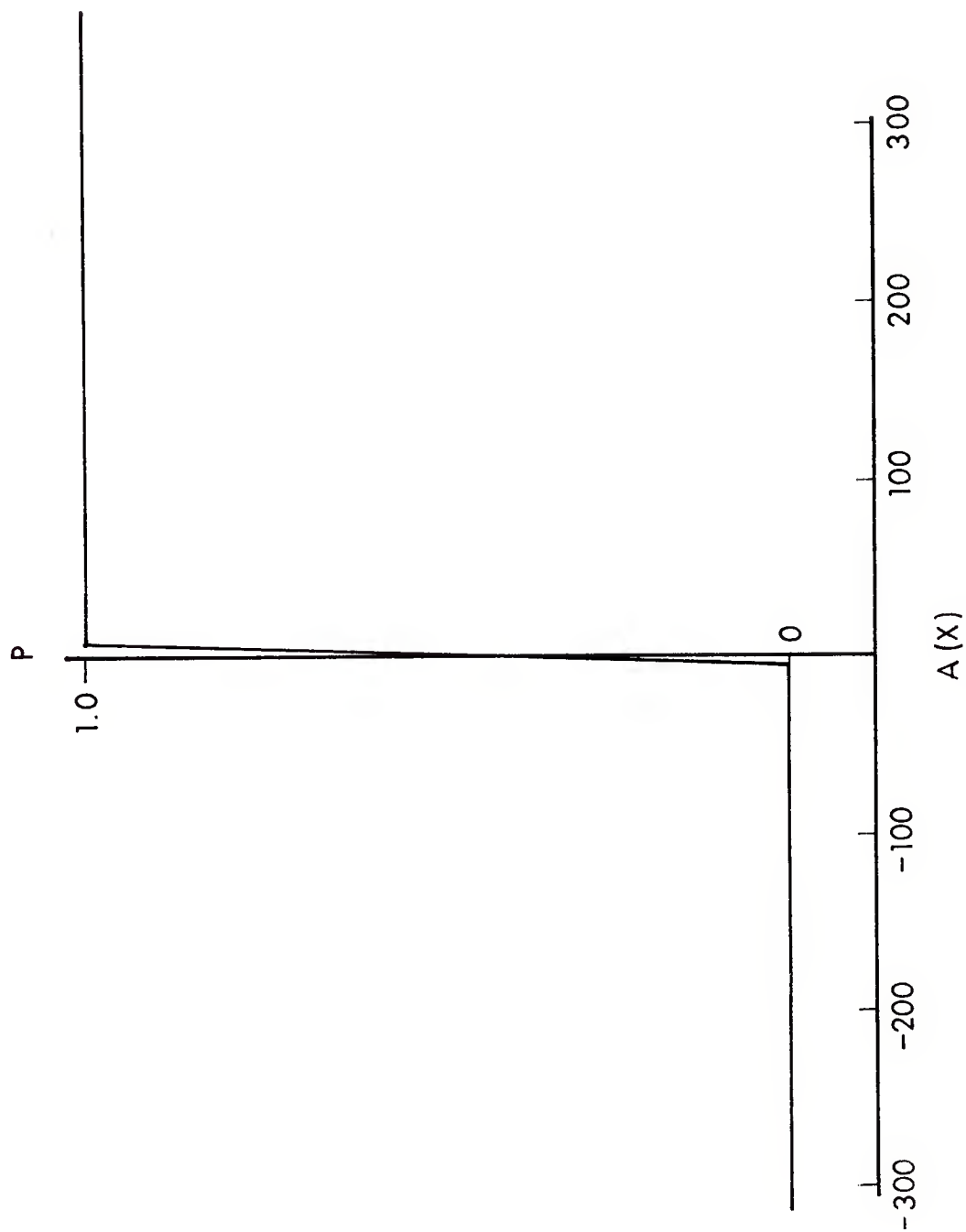
$A(x) = A_0 + A_1 X + A_2 X^2 + \ldots + A_n X^n$

Figure 2. A Graphical Representation of the Logistic Function for 35 Cases.

Regression coefficients for the initial state inputs with respect to outcomes are provided in Table 4, again assuming the same simple linear form of the variables that was used to predict attacker win or loss.

The results, presented in Table 4, indicate that exposure (E) of the defender and time-frame (T) are most important to response. These results substantiate those presented in the PER and correlation methodologies.

TABLE 4   REGRESSION ON 35 AMSWAG GAMES

	CONSTANT	E	M	T	P	C
TEOG	62.75	24.78	-2.19	11.38	-3.05	-0.40
RED/BLUE (VEHICLES)	3.92	- 0.28	-0.01	-0.44	-0.11	-0.08
RED/BLUE (PERSONNEL)	4.38	-0.24	-0.01	-0.44	-0.09	-0.07

The signs of the coefficients are also informative. For example, consider either final force ratio for vehicles or personnel (attacker/ defender). As the index of exposure increases (i.e., the defender is in a more defilade position), it has a negative or detrimental affect on the final force ratio. An analogous situation applies to TEOG. Since both coefficients for exposure (E) and time-frame (T) are positive, an increase of their indices would increase the time which a game lasts.

The E_{RMS} (error root mean squared)value for TEOG and the final force ratios are listed below. It should be noted here that TEOG varied in ten-second increments over a range of [0,180]. These E_{RMS} values for the 3 outcomes, represent good fits.

	E_{RMS}
TEOG	15.56
VEHICLES (RED/BLUE)	0.12
PERSONNEL (RED/BLUE)	0.16

2.2.2.4 Summary (Phase 1 Study). It has been shown that the methodologies of PER and regression can be applied to variables describing the initial state for the 35 games studied in predicting outcomes of interest.

PER and the logistic function were useful tools for predicting win or loss. PER provided information gain and relative information gain for each variable. The logistic function provided a predictor for the probability of attacker win over the entire "state-space."

Standard linear regression techniques were useful (when the dependent variable is continuous) in predicting the outcomes of game length and final force ratios from the initial state.

2.2.3 Application of Approach to an Expanded Data Base (155 Games). As mentioned in previous sections, 120 additional AMSWAG runs from the Engineering Study Phase II were added to the data base. The total number of games then under study was 155.

The same approach applied previously is used here, that is, the methodologies of PER, logistic function, and linear regression. Since the data base has grown substantially, cluster analysis has been added as a tool for analyses. Its application will be described subsequently.

The variables of initial engagement range, visibility, and initial force ratio were present in the 120 new cases and so the state vectors for the original 35 games were expanded to include them as well.

Initial engagement range (I) was preset at 2.5, 2.0, 1.5, or 1.0 kilometers. Initial force ratio (IFR) was the ratio of the attacking force to the defending force. Visibility was either restricted or unrestricted and indexed 1 or 2 respectively. Criterion for a restricted visibility was that visual acquisition was limited to a range of 1.0 kilometers.

2.2.3.1 PER and Correlation Study (Combined Study). Table 5 lists $P(Att_w)$, the independent input variables and their associated values of E_x (Information gain of a variable x), and R_x (Relative information gain).

It can be seen that the variables of minefield density (M), initial engagement range (I), preparation time (P), initial force ratio (IFR), time-frame (T), and defender's exposure (E) have the highest associated values of R_x. Neither countermeasure (C) nor visibility (V) seem to be strongly related to outcome.

Significant differences in R_x can be observed for certain variables between the original and expanded data base. For example, C had an R_x value of .34 for the 35 game study but of .05 for all 155 cases. This drop is due to the lack of variation present in C in the expanded base, which prevents it from being a good predictor of outcome.

In conjunction with PER, correlations were obtained for all input variables. Table 6 gives the correlation matrix of the eight

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TABLE 5   PER RESULTS ON COMBINED 155 GAMES

	$P(Att_w)$	E_x	R_x
EXPOSURE (E)	0.66	0.14	0.31
MINEFIELD DENSITY (M)	0.66	0.27	0.59
TIME-FRAME (T)	0.66	0.14	0.32
COUNTERMEASURE (C)	0.66	0.02	0.05
VISIBILITY (V)	0.66	0.04	0.09
INITIAL FORCE RATIO (IFR)	0.66	0.16	0.35
INITIAL RANGE (I)	0.66	0.23	0.51
PREPARATION TIME (P)	0.66	0.22	0.50

TABLE 6   CORRELATION ON 8 INPUT VARIABLES
155 CASE COMBINED STUDY

E	M	T	C	V	IFR	I	P
1.0	.01	.05	-.05	-.21	-.03	.00	.04
	1.00	-.07	.18	.03	-.07	-.08	.91
		1.00	-.28	-.34	-.40	.43	-.14
			1.00	.08	-.16	-.17	.24
				1.00	.04	-.09	.04
					1.00	.44	-.13
						1.00	-.16
							1.00

initial state variables used. It can be seen that a high correlation exists between minefield density (M) and preparation time (P). It was determined that the exclusion of one of these variables would not affect the results. The variable minefield density was excluded, thus reducing the initial state space to seven variables.

2.2.3.2 Logistic Function (Combined Study). The parameters for the logistic function in which each variable entered the polynomial linearly were also obtained for all of the 155 engagements in the combined study. The polynomial is given below:

$$A(x) = 5.9405 - .6022(E) - .9577(T) + .1741(C)$$

$$- .9774(V) + 1.3981(IFR) - .1419(I) - .9718(P).$$

This function, $A(x)$, was used to calculate P(probability of an attacker win) for the 155 cases. There were 27 misclassifications observed, where a misclassification is defined as: any game where the absolute value of the predicted probability of win minus the actual outcome, a 1 or 0, is greater than 0.5. More simply stated, a game is classified as an attacker win if the logistics function prediction of the probability of attacker win exceeds 0.5; otherwise, it is classified as a defender win.

It may be noted that there is a 17.4% probability of misclassification associated with this "overall" logistic function for all 155 cases. This result may reflect the inability of a single model, particularly a non-phenomenological one, to make accurate predictions of outcome over the large regions of a high dimensional space. In later sections of this report cluster analysis and further modeling within clusters are shown to reduce the 27 misclassifications to three.

2.2.3.3 Cluster Analysis (Combined Study). Since the data base has increased from 35 AMSWAG cases to 155, and the dimensionality of the state space has increased from five to seven, the ability of a single model to make accurate predictions over the entire state space becomes limited. It would be useful to derive more locally restricted yet more accurate predictive models. Cluster analyses is a set of mathematical methods for separating large numbers of vectors into smaller compact subsets called clusters. In the particular clustering method used in this study [18], [19], input or "state" vectors "close" to one another in a Euclidean Distance sense are grouped into clusters. The use of parameters, to control the lumping and splitting of clusters on successive iterations, helps to attain the desired degree of cluster separation and cluster compactness.

Using this technique, clusters were obtained for the 155 case study. An a priori probability P, of an attacker win is estimated by the relative percentage of attacker wins in that cluster. Table 7 lists clusters according to class (Attacker Win or Loss) and gives the P (probability of attacker win) within each cluster.

TABLE 7   CLUSTER MEMBERSHIP TABLE

CLUSTER #	(ATTACKER)		P (PROB. ATTACKER WIN)
	LOSS	WIN	
1	12	15	0.56
2	0	4	1.0
3	0	4	1.0
4	0	4	1.0
5	1	16	0.94
6	4	0	0.0
7	4	0	0.0
8	0	5	1.0
9	11	12	0.52
10	5	23	0.82
.11	16	11	0.41
12	0	8	1.0

It can be seen from Table 7 that clusters 1, 9, 10, and 11 are candidates for further modeling, since all other clusters have little variability of outcome occurring within them, and hence, require no formal modeling in order to make future predictions of outcome for games whose input states reside in those clusters.

The methodologies of PER and linear correlation were applied to the games within each cluster. These methodologies assisted in choosing the best initial state variables (within clusters) to be included in the appropriate regression model. That is, in different regions of the state space (i.e., in different clusters) different variables are useful in predicting outcome and should be utilized in the more localized models.

By the use of the above methodologies the variables of exposure (E), time-frame (T), initial engagement range (I), and preparation time (P) were chosen for the linear regression on cluster 1. Time-frame (T), countermeasure (C), initial force ratio (IFR), and initial engagement range (I) were used to model clusters 9, 10, and 11. Table 8 associates the overall logistic function and each cluster with its linear coefficients and ERMS (error root mean squared).

TABLE 8 REGRESSION COEFFICIENTS FOR "OVERALL" LOGISTIC FUNCTION AND INDIVIDUAL CLUSTER REGRESSION MODELS

VARIABLE COEFFICIENTS

Logistic/Cluster Models	Constant	E	T	C	IFR	I	P	V	ERMS
Overall Logistic Function	5.94	-0.60	-0.96	0.17	1.40	-0.14	0.97	0.98	*
Cluster #1**	2.34	-0.20	-0.49	N/A	N/A	-0.07	-0.05	N/A	0.25
Cluster #9	-2.60	N/A	0.69	0.84	0.28	-0.00	-0.00	N/A	0.22
Cluster #10	1.59	N/A	0.06	-0.88	0.03	-0.00	-0.00	N/A	0.20
Cluster #11	-1.08	N/A	0.04	0.14	0.30	-0.00	-0.00	N/A	0.41

NOTE:

N/A: Not applicable
*: Not obtained

** For example, the model for Cluster #1 is: $P(ATTW) = 2.34 - .20E - .49T - .07I - .05P$

Referring to Table 8 we see that time-frame (T) and exposure (E) contribute most to the model for cluster #1. The model for cluster #9 uses time-frame (T), countermeasure (C), and initial force ratio (IFR). The model for cluster #10 heavily depends on countermeasure (C). It is interesting to note that the coefficients of countermeasure (C) for cluster #9 and cluster #10 have opposite signs; hence any change in countermeasure (C) affects outcomes in these clusters in opposite ways.

The model for cluster #11 is the most unique. It makes its predictions mainly on initial force ratio (IFR) and countermeasure (C).

2.2.3.3.1 Results of Cluster Analysis. It has been shown in Table 7 that cluster membership can be used to provide an estimate of probability of an attacker win. This is beneficial when a known initial state vector falls within a cluster containing either all wins or all losses in that modeling within that cluster was not necessary.

Earlier it was observed that when a single response surface (logistic function) was derived to represent the outcomes for state vectors for all 155 games in the Engineering Study data base, that model produced

27 misclassifications. By using models developed for specific clusters, the misclassifications have been reduced to three, i.e., a misclassification rate of less than 2%. These were produced by the linear regression predictions for single games in clusters 10 and 11 and the single loss vector observed in cluster #5.

Listed in Table 9 are the state variables for the three cases that were misclassified after modeling within individual clusters was performed.

TABLE 9   CASES CONSIDERED ANOMALIES AFTER CLUSTER ANALYSIS

CASE#	CLUSTER#	E	T	C	VIS	IFR	I	P	%LOSS BLUE	% LOSS RED
02B	5	1.0	2.0	2.0	2.0	2.6	2.0	1.0	59	60
06A	10	4.0	2.0	1.0	2.0	3.7	2.0	1.0	59	60
34P	11	4.0	2.0	1.0	2.0	6.2	2.0	4.0	51	60

It should be noted that all three of these vectors exhibit two similar characteristics which may provide a possible explanation for their misclassification. The first being that all three games should be considered close with respect to percent losses. That is, while the arbitrary "loss" criteria in AMSWAG defined a definite winner and loser, the outcome was, in actuality, very much in doubt. A second explanation appears to be related to an initial engagement range (I) of 2.0 kilometers. It should be noted that the three anomalies are from the Engineering Phase II study. Cases 02B, 06A, and 34P are similar in that an attacker loss is expected but the associated within cluster model predicts an attacker win. Each misclassified case listed in Table 10 along with the others in its structured group of cases as evaluated in the Engineer Study.

TABLE 10   CASES CONSIDERED ANOMALIES WITHIN STRUCTURED DATA

CASE#	CLUSTER#	E	T	C	VIS	IFR	I	P	%LOSS BLUE	%LOSS RED
01B	5	1.0	2.0	2.0	2.0	2.6	2.5	1.0	60	44
02B*	5	1.0	2.0	2.0	2.0	2.6	2.0	1.0	59	60
03B	5	1.0	2.0	2.0	2.0	2.6	1.5	1.0	60	33
04B	5	1.0	2.0	2.0	2.0	2.6	1.0	1.0	60	30
05A	10	4.0	2.0	1.0	2.0	3.7	2.5	1.0	60	39
06A*	10	4.0	2.0	1.0	2.0	3.7	2.0	1.0	59	60
07A	10	4.0	2.0	1.0	2.0	3.7	1.5	1.0	60	33
08A	10	4.0	2.0	1.0	2.0	3.7	1.0	1.0	60	28

TABLE 10 (continued)

CASE#	CLUSTER#	E	T	C	VIS	IFR	I	P	%LOSS BLUE	%LOSS RED
33P	11	4.0	2.0	1.0	2.0	6.2	2.5	1.0	60	54
34P*	11	4.0	2.0	1.0	2.0	6.2	2.0	1.0	51	60
35P	11	4.0	2.0	1.0	2.0	6.2	1.5	1.0	60	53
36P	11	4.0	2.0	1.0	2.0	6.2	1.0	1.0	60	58

NOTE: Cases designated with * are considered anomalies.

Within the structured data groups, it can be seen that the 3 misclassifications are produced at an initial engagement range of 2.0 kilometers. These anomalies are similar to those found when initial engagement range is compared to effectiveness measures within the Engineering Phase II study.[1] It was found in that study that, in general, as initial engagement range decreases, so also does the Blue forces' effectiveness. However, it was also stated that all Blue effectiveness measures "sawtoothed" at 2.0 kilometers, yielding the results which we see in our cluster modeling.

3. SUMMARY/CONCLUSIONS/RECOMMENDATIONS

In this report, it has been shown that the methodologies of PER, linear and non-linear regression, and later with an expanded data base, cluster analyses could be used to predict probabilities of win and other outcomes of interest. It is important to remember that this was accomplished based only upon the initial state of a wargame and, of course, knowledge concerning simulation outcomes.

PER and linear correlation were instrumental in providing information gain concerning input variables and their relationships to the outcomes of interest.

The logistic function was shown to be powerful in predicting probability of win, especially with the initial 35 case study, where it was perfect. Later, when the data base was expanded to 155 cases, cluster analyses enhanced the accuracy available from the regular regression approach. Modeling of individual clusters by linear regression techniques provided better models of more evenly split clusters (clusters which included both wins and losses).

Design and modeling of this type provides a basis for: quantitatively determining the significance of individual factors and interactions on outcomes of interest in a wargame; providing a method for critically comparing the results of studies and influencing developmental efforts.

The approach provides a way for more efficiently using the results of a small number of runs and a consistent method for analyzing a large number of runs.

3.1 Conclusions and Thoughts on Future Work.

This report briefly reviewed how several techniques and methods had been used to "model" complex systems and/or simulations. The specific objective of this report was to apply these techniques to the representation and analysis of results obtained with AMSWAG, a combat simulation frequently used at AMSAA. The benefits and uses of the approach were outlined and specific results were obtained for 35 and 155 AMSWAG cases.

These runs had been performed to support two Engineer Studies. Our analyses were meant to demonstrate the methodology, with the model runs which were currently available. No attempt was made to structure and perform additional runs due to time and cost constraints.

When reviewing the available runs it became apparent that they had been structured based upon the "one at a time" approach. With this method, all input variables (or factors) except one are held constant while the remaining factor is cycled through its possible values. The process is then repeated for each variable. This is done in order to gain some insight into the effect on the responses of interest, of changing each variable individually.

There are several limitations to this approach. First, each group of runs provides information only concerning the effect of the one variable which is being altered. If instead, all the components in the vector describing the input to the model had been varied simultaneously, information concerning the effects of other factors could have been extracted from the runs. Further, the one at a time approach is able to assess the effects of each variable assuming that no interactions exist, and is not able to consider the interactions themselves. (An interaction is defined to exist between two factors when the change in response due to a change in the level of one factor depends upon the level of the other factor). For example, the diagram below indicates the presence of an interaction between factors A and B.

Level of Factor B

OUTPUT

b_0

b_1

Factor A

a_0 a_1
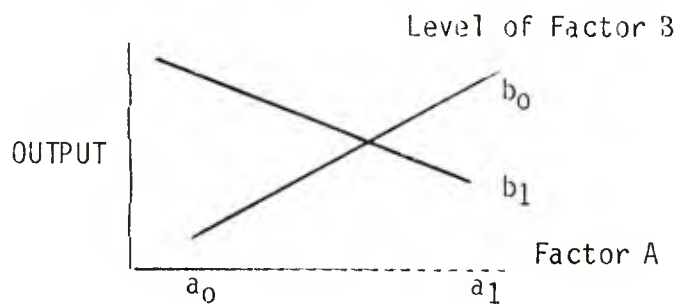
In the above example, the main effect of A is of little use in prediction and we may wish only to estimate the interaction AB. If we are unable to assess interactions, as in the case where the "one at a time" approach was used, the study could derive erroneous conclusions.

A factorial experimental design approach to structuring the runs for such an analysis overcomes the limitations of the "one at a time" method. With this approach information concerning the contributions of interactions and main effects of factors can be obtained and tested (with equal precision) for their significance -- a procedure which has in the past been assessed only subjectively. Thus, the factorial procedure would add objectivity to the portrayal of study results and the comparison of results from study to study.

Future endeavors in this area will concentrate on:

(1) continuous updating of the data base and revision of predictive models;

(2) participation in the design of sets of runs to be used in future AMSWAG efforts.

(3) the development of models which predict outcome based upon the "state" of the game at times other than t_0; i.e., dynamic state.

There are ways in which the applicability and efficiency of the data base might be improved. We will now discuss several possibilities related to (1) and (2) above.

An existing "state space" model can be improved and broadened by the inclusion of input/output from additional model runs. When the results of a new study are added, predictive models for the entire data base can be regenerated rapidly. Reclustering and, if needed, new models for the clusters affected can also be regenerated efficiently.

A limitation which arises in the use of this type of surrogate modeling occurs when the structure of the simulation being modeled undergoes change. That is, as changes are made to AMSWAG, the surrogate models must be updated and revalidated. Revalidation must be undertaken when cluster membership is significantly affected and/or new clusters are developed. The revalidation methods would be similar to those used in the text of this report, that is, a comparison is made of the surrogate's estimates with the actual model results.

Examples of this situation could arise in cases where force constitution has changed or the "initial state" has been altered to the point where no part of the existing data base is representative.

Concerning item (3), when considering the state as a function of time we wish to construct a response surface which gives the probability of a win given the game is in some state at time t, $(P(W/S(t)))$, where $P(W/S(t))$ is the probability of a win given that $S(t)$ is the state of the game at time t. With such a probability distribution an objective would be to establish regions or partitions in the $S(t)$ space for which $P(W/S(t)) = k$, for $k = 0, .1, .2, .3, \ldots, 1.0$. The costs associated

with the new developments or changes in tactics which are required to go from one region to another with a higher probability of win could then be assessed and give direction to future efforts.

Significant work has already been undertaken in this area. The results of these efforts will soon be reported.

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APPENDIX

PER METHODOLOGY

The PER methodology is a way to assess the ability of an
independent variable to predict a bivariate response (it is analogous
to the role played by correlations when both independent and dependent
variables are continuous). For example, one response to be concerned
with in the war game analyses is the outcome of the game. This outcome
may take on a value of 1 or 0 depending on whether the attacking force
wins or loses.

PER is based on certain information theoretic concepts and is
in fact an acronym. P stands for the a priori probability of outcome 1
(probability of the attacker winning).

$$P = \frac{\text{Number of outcomes with response 1}}{\text{total number of outcomes}}$$

= relative percent of games the attacker wins. E_x is the
average information gain of a variable x. It is the amount by which one
would change his or her estimate of game outcome, given a value of x. The
information gain of a perfect predictor x is given by:

$$E_x = P(1-P) + (1-P)P \quad \text{or}$$

$$E_x = 2P(1-P)$$

This may be seen in the following way. Given no value of X, the
best estimate of the probability of a "1" outcome is P, the a priori proba-
bility of 1. Given the value of x a perfect predictor, one would move his
prediction from P to 1 (a distance of 1-P) for all 1's which occur P percent
of the time. Similarly, the prediction would move from P to "0" (a distance
of P) for all zero outcomes, which occur (1-P) percent of the time. The
average distance the prediction is moved, given a value of x, then is
seen to be 2P(1-P).

When computing E_x, a partitioning must be established on the
range of values taken on by x. If x is a continuous variable such a par-
titioning can be obtained, for example, by dividing the range into a
number (usually 5-10) of equal width lines. If x is discrete and takes on
only a small number of values, bins may be defined on the range of x,
which can be indexed from 1 to K. Within a bin we can count the number of
observations of x which were associated with outcomes of "1". The
probability of a "1" outcome occurring in the jth bin, j = 1, 2, 3, ..., k,
is:

$$P("1"/j) = \frac{\text{Number of "1" outcomes associated with x value in bin j}}{\text{Number of outcomes in bin j}}$$

The frequency with which observations fall into the jth bin is:

$$f_j = \frac{\text{Number of observations of x in the jth bin}}{\text{Total number of observations of x}}$$

hence,

$$E_x = \sum_{j=1}^{k} f_j \; |P-P("1"/j)|$$

It should be noted that information gain, E_x, depends upon the probability of the attacker winning P. Previously, it was shown that E_x for a perfect predictor x was $2P(1-P)$. Thus, for any variable x, its E_x is limited by the range: $0 \leq E_x \leq 2P(1-P)$.

The relative information gain, R in the PER acronym is a normalized version of E_x which removes the dependency of P.

For a variable x, the relative information gain R_x, is defined as:

$$R_x = \frac{E_x}{2P(1-P)}$$

R_x represents the predictive capability of the variable x relative to the perfect predictor P. It should be noted that $0 \leq R_x \leq 1$ and the greater the value of R_x the more useful is x alone at making predictions concerning outcome.

A few other comments should be made concerning PER. First, in those cases when outcome is a bivariate one, the use of a correlation coefficient is not appropriate in determining the degree of relationship which exists between any two variables. PER, therefore, provides an attractive alternative. Secondly, it is well known that correlation measures the degree of linear relationship that exists between two variables. At times, it is probable that two variables will exhibit a low or zero correlation, and yet are perfectly, but non-linearly, related. Therefore, if a low correlation with outcome is observed with respect to an independent variable, it could wrongly be excluded from further analyses. PER has no such limitation; it identifies such a variable as a useful predictor of outcome.

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